Math 43 Midterm 3 Review

In addition to the following review questions, you must be able to solve any of the questions from the 3D Lines & Planes handout.

[1] Consider the vector $\vec{m} = \langle -2, -4 \rangle$, and the vector $\vec{n}$ with direction angle $\frac{2\pi}{3}$ such that $\|\vec{n}\| = 8$.

[a] Find a unit vector perpendicular to $\vec{m}$. (Do **NOT** use decimal approximations.)

[b] Find the direction angle of $\vec{m}$. (Your answer should be in radians, rounded to 2 decimal places.)

[c] Write $2\vec{n} - \vec{m}$ as a linear combination of $\vec{i}$ and $\vec{j}$. (Do **NOT** use decimal approximations.)

[2] Consider the vectors $\vec{f} = 2\vec{j} - 3\vec{k}$ and $\vec{g} = -\vec{i} - 3\vec{j} + 4\vec{k}$.

[a] Find the angle between $\vec{f}$ and $\vec{g}$. (Your answer should be in radians, rounded to 2 decimal places.)

[b] Find a unit vector perpendicular to both $\vec{f}$ and $\vec{g}$. (Do **NOT** use decimal approximations.)

[c] Write $\vec{f}$ as the sum of 2 vectors, one parallel to $\vec{g}$ and one perpendicular to $\vec{g}$. (Do **NOT** use decimal approximations.)

[d] If the terminal point of $\vec{g}$ is $(-7, 4, -8)$, find the initial point.

[e] If $\vec{h} = a\vec{i} + b\vec{j} - 5\vec{k}$ is parallel to $\vec{g}$, find the values of a and b.

[f] If $\vec{e} = 7\vec{i} + c\vec{j} - 5\vec{k}$ is perpendicular to $\vec{g}$, find the value of c.

[3] Let P be the point $(-5, -2, 3)$. Let Q be the point $(3, 2, -1)$. Let R be the point $(-3, 4, -2)$.
Let $\vec{u}$ be the vector with initial point R and terminal point Q.
Let $\vec{w}$ be the vector with initial point P and terminal point R.

[a] In which octant is R ?

[b] If you start at point P, move 2 units down, 4 units back, and 6 units to the right, find the co-ordinates of your ending point.

[c] Write $\vec{u}$ in component form.

[d] Write $\vec{w}$ as a linear combination of $\vec{i}$, $\vec{j}$ and $\vec{k}$.

[e] Find the magnitude of $\vec{w}$. (Do **NOT** use decimal approximations.)

[f] Find a unit vector in the opposite direction as $\vec{w}$. (Do **NOT** use decimal approximations.)

[g] Find a vector of magnitude 6 in the same direction as $\vec{u}$. (Do **NOT** use decimal approximations.)

[h] If $\|\vec{v}\| = 3$, and the angle between $\vec{u}$ and $\vec{v}$ is 2 radians, find $\vec{u} \cdot \vec{v}$. (Round your answer to 2 decimal places.)

[i] If $\|\vec{v}\| = 3$, and the angle between $\vec{u}$ and $\vec{v}$ is 2 radians, find the magnitude of $\vec{u} \times \vec{v}$. (Round your answer to 2 decimal places.)

[j] Find the area of triangle PQR. (Do **NOT** use decimal approximations.)

[k] Find $\angle QRP$. (Round your answer to 2 decimal places.)

[l] If a force represented by the vector $4\vec{i} - 5\vec{k}$ is applied to an object as it moves from Q to P, find the work done.

[m] Find the general equation of the plane passing through P, Q and R.

[n] Find parametric equations for the line which passes through P and is also parallel to $\vec{u}$.

[o] Find symmetric equations for the line which passes through Q and is also perpendicular to the plane $-2x - 3y + z = 9$.

[p] Find the equation of the sphere with P and Q as endpoints of a diameter.

[4] Which octant or octants contain all points (x, y, z) where $xz < 0$?

[5] Consider the sphere $x^2 + y^2 + z^2 - 4x + 6y + 10z + 29 = 0$.

[a] Find the center and radius of the sphere.

[a] Find the equations of the $xy-$, $xz-$ and $yz-$traces of the sphere, and describe each trace.

Math 43 Midterm 3 Review Answers

[1] [a] $< \frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}} >$ *or* $< -\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} >$

[b] ≈ 4.25 radians

[c] $-6\vec{i} + (4 + 8\sqrt{3})\vec{j}$

[2] [a] ≈ 2.94 radians

[b] $< -\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}} >$ *or* $< \frac{1}{\sqrt{14}}, -\frac{3}{\sqrt{14}}, -\frac{2}{\sqrt{14}} >$

[c] $< \frac{9}{13}, \frac{27}{13}, -\frac{36}{13} > + < -\frac{9}{13}, -\frac{1}{13}, -\frac{3}{13} >$

[d] $(-6, 7, -12)$

[e] $a = \frac{5}{4}, \quad b = \frac{15}{4}$

[f] -9

[3] [a] octant 6

[b] $(-9, 4, 1)$

[c] $< 6, -2, 1 >$

[d] $2\vec{i} + 6\vec{j} - 5\vec{k}$

[e] $\sqrt{65}$

[f] $< -\frac{2}{\sqrt{65}}, -\frac{6}{\sqrt{65}}, \frac{5}{\sqrt{65}} >$

[g] $< \frac{36}{\sqrt{41}}, -\frac{12}{\sqrt{41}}, \frac{6}{\sqrt{41}} >$

[h] ≈ -7.99

[i] ≈ 17.47

[j] $2\sqrt{165}$

[k] ≈ 1.47 radians

[l] -52

[m] $x + 8y + 10z - 9 = 0$

[n] $x = -5 + 6t, \quad y = -2 - 2t, \quad z = 3 + t$ **OTHER ANSWERS POSSIBLE**

[o] $\frac{x-3}{2} = \frac{y-2}{3} = -z - 1$ **OTHER ANSWERS POSSIBLE**

[p] $(x+1)^2 + y^2 + (z-1)^2 = 24$

[4] $2, 3, 5, 8$

[5] [a] center $= (2, -3, -5)$, radius $= 3$

[b] no $xy-$trace

$xz-$trace is point $(2, 0, -5)$

$yz-$trace has equation $(y+3)^2 + (z+5)^2 = 5$ [circle in $yz-$plane, center $= (0, -3, -5)$, radius $= \sqrt{5}$]